2019 4th International Workshop on Materials Engineering and Computer Sciences (IWMECS 2019)

Research on Mathematical Model of Drink-driving Problem

Cheng Hongping, Shao Zhiwei

Xi’an Eurasia University, Xi’an, China

Keywords: drink-driving; safety time; mathematical model

Abstract: According to the changes in the absorption, metabolism and excretion of alcohol in human body after drinking, this paper adopts scientific experiment method and combines mathematical model analysis, and concludes the function equation of alcohol in blood changing in accordance with different amount of alcohol, providing safety driving time that can be chosen by drivers after drinking.

1. Research Questions

People often say, “Don't drink and drive!” However, sometimes friends go out to party, and they can’t avoid drinking a few cups at the dinner table. According to data from World Health Organization, in developing countries, one person dies from a traffic accident related to drinking every 33 minutes. Therefore, how to deal with the problem of drinking drive is closely related to the safety of each citizen. When a person drinks, if he can scientifically predict the content of alcohol in his own blood at different times, then he can choose the right time to drive, which will not violate traffic regulations, but also avoid traffic accidents.

According to the national standard for the Threshold and Inspection of Blood and Expiratory Alcohol Content of Vehicle Drivers issued by General Administration of Quality Supervision, Inspection and Quarantine of the People’s Republic of China, the alcohol content in the blood of vehicle drivers greater than or equal to 20 mg/100 ml, less than 80 mg/100 m belongs to drinking drive. (The original standard is less than 100 mg/100 ml). The alcohol content in the blood greater than or equal to 80 mg/100 ml is drunk driving. (The original standard is greater than or equal to 100 mg/100 ml). In addition, medical research shows that when 40 mg of alcohol is contained in 100 ml of blood, there is clumsy movement; 60 mg, chattering; 80 mg, emotional and unresponsive; 120 mg, tired and sleepy; 200 mg, loss of control; 400 mg, coma; 600 mg, death.

In view of this practical problem, based on the corresponding data obtained by scientific experiments, under the influence of time and the factors of drinking methods, this paper establishes the corresponding mathematical model to obtain the change of alcohol concentration in the blood, providing safety driving time that can be chosen by drivers after drinking.

2. Mathematical Model

The establishment and solution of model is presented. In general, the volume of human gastrointestinal tract and the volume of body fluid are constant. The process of alcohol transfer in the human body is divided into three processes: alcohol, gastrointestinal, body fluid, and in vitro. When people drink alcohol, alcohol (ie, ethanol) first enters stomach, then absorbed into the bloodstream, and as the blood flows through the organs and tissues of the body, it is continuously absorbed, metabolized, and excreted. The process can be simply shown in Figure 1.

According to the different ways of drinking in daily life, such as fast drinking in a short period of time, slow drinking in a certain period of time, quickly drinking the same amount of wine at regular intervals, different mathematical models are established, so that the equations of the changes in alcohol concentration in body fluid over time in different situations.

  DOI: 10.25236/iwmecs.2019.089

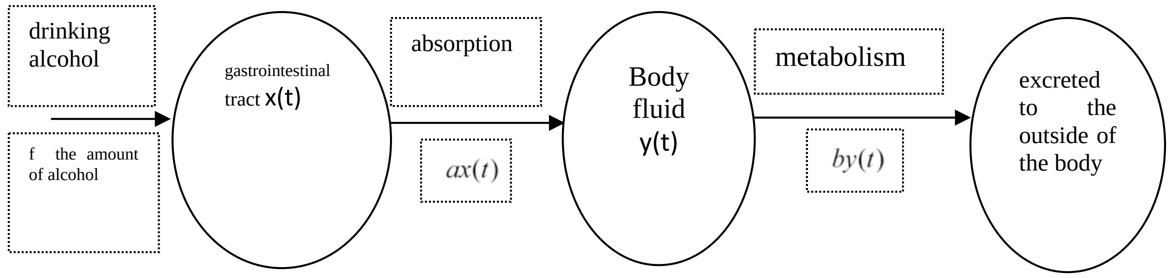

Figure 1 The transfer process of alcohol

2.1 The alcohol concentration in body fluid while fast drinking in a short period of time

After drinking quickly, the alcohol first enters the gastrointestinal tract and is absorbed. Based on the hypothesis that the speed of the transfer from the gastrointestinal to the body fluid is proportional to the amount of alcohol, $\frac{dx}{dt} = -ax(t)$ can be obtained. Then it reaches the body fluid, and is excreted as part of the metabolism. From the hypothesis that the speed of excretion is proportional to the amount of alcohol in body fluid, $\frac{dy}{dx} = ax(x) - by(t)$ can be obtained. So there is Model I.

$$\begin{cases} \frac{dx}{dt} = -ax(t) \\ \frac{dy}{dx} = ax(x) - by(t) \\ x(0) = f \\ y(0) = 0 \end{cases}$$

The solution of differential equations can be obtained.

$$x(t) = fe^{-at} \qquad y(t) = \frac{af}{a-b}(e^{-bt} - e^{-at})$$

So the alcohol content in body fluid is shown in Equation (1).

$$c(t) = \frac{y(t)}{v} = \frac{af}{v(a-b)}(e^{-bt} - e^{-at}) \qquad (1)$$

In Equation (1), t (hour): time, $x(t)$: the amount of alcohol in the gastrointestinal tract (mg), $y(t)$: the amount of alcohol in the body fluid (mg), a: the transfer coefficient of alcohol from the gastrointestinal to the body fluid, b: the discharge coefficient of alcohol from the body fluid to the outside of the body, v: volume of body fluid (100 ml), f: the amount of alcohol (mg) from the mouth into the gastrointestinal tract, $c(t)$: the amount of alcohol in the body fluid in t time (mg/100 ml).

2.2 The alcohol concentration in body fluid while slow drinking in a certain period of time (τ)

$\frac{f}{\tau}$ is the rate of drinking, considered as two stages. The first stage is $0 \le t \le \tau$: always drinking; the second stage is $t > \tau$: no longer drinking. ModelII-1 (drinking at a constant speed) can be obtained from the analysis.

ModelII-1: $0 \le t \le \tau$ (drinking at a constant speed)

$$\begin{cases} \dfrac{dx^{(1)}}{dt} = \dfrac{f}{\tau} - ax^{(1)}(t) \\ \dfrac{dy^{(1)}}{dt} = ax^{(1)}(t) - by^{(1)}(t) \\ x^{(1)}(0) = 0 \\ y^{(1)} = 0 \end{cases}$$

The solution of the equations is given.

$$x^{(1)} = \frac{f}{\tau a}(1-e^{-at}) \qquad y^{(1)}(t) = \frac{af}{\tau(a-b)}(\frac{1-e^{-bt}}{b} - \frac{1-e^{-at}}{a})$$

So the alcohol content in body fluid is shown in Equation (2).

$$c(t) = \frac{y^{(1)}(t)}{v} = \frac{af}{v\tau(a-b)}(\frac{1-e^{-bt}}{b} - \frac{1-e^{-at}}{a}) \qquad (0 \le t \le \tau) \qquad (2)$$

Model II-2: $t > \tau$ (after drinking to t time)

$$\begin{cases} \dfrac{dx^{(2)}}{dt} = -ax^{(2)}(t) \\ \dfrac{dy^{(2)}}{dt} = -ax^{(2)}(t) - by^{(2)}(t) \\ x^{(2)}(\tau) = x^{(1)}(\tau) \\ y^{(2)}(\tau) = y^{(1)}(\tau) \end{cases}$$

The solution of the equations is given.

$$x^{(2)}(t) = \frac{f}{a\tau}(1-e^{a\tau})e^{-a(t-\tau)} \qquad y^{(2)}(t) = \frac{af}{\tau(a-b)}[\frac{1-e^{-b\tau}}{b}e^{-b(t-\tau)} - \frac{1-e^{-a\tau}}{a}e^{-a(t-\tau)}]$$

So the alcohol content in body fluid is shown in Equation (3).

$$c(t) = \frac{y^{(2)}(t)}{v} = \frac{af}{v\tau(a-b)}[\frac{1-e^{-b\tau}}{b}e^{-b(t-\tau)} - \frac{1-e^{-a\tau}}{a}e^{-a(t-\tau)}] \qquad (t > \tau) \quad (3)$$

2.3 The alcohol concentration in body fluid while quickly drinking the same amount of alcohol (For each time, the amount of alcohol is f) at regular intervals (T hour)

The moment of the i time drinking is used as the starting time. After t hours, the amount of alcohol in the gastrointestinal tract is recorded as $x_i(t)$, and the amount of alcohol in the body fluid as $y_i(t)$.

Model III can be obtained.

$$i = 1 \qquad \begin{cases} \dfrac{dx_1}{dt} = -ax_1(t) \\ \dfrac{dy_1}{dt} = ax_1(t) - by_1(t) \\ x_1(0) = f \\ y_1(0) = 0 \end{cases}$$

$$i = 2 \qquad \begin{cases} \dfrac{dx_2}{dt} = -ax_2(t) \\ \dfrac{dy_2}{dt} = ax_2(t) - by_2(t) \\ x_2(0) = x_1(T) + f \\ y_2(0) = y_1(T) \end{cases}$$

....

$$i = n \qquad \begin{cases} \dfrac{dx_n}{dt} = -ax_n(t) \\ \dfrac{dy_n}{dt} = ax_n(t) - by_n(t) \\ x_n(0) = x_{n-1}(T) + f \\ y_n(0) = y_{n-1}(T) \end{cases}$$

The solutions of each equations are given.

$$\begin{cases} x_1(t) = fe^{-at} \\ y_1(t) = \dfrac{af}{a-b}(e^{-bt} - e^{-at}) \end{cases} \quad (0 \le t \le T)$$

$$\begin{cases} x_2(t) = x_1(t) + x_1(T)e^{-at} = f(1+e^{-aT})e^{-at} \\ y_2(t) = y_1(t) + y_1(T)e^{-at} = \dfrac{af}{a-b}[(1+e^{-bT})e^{-bt} - (1+e^{-aT})e^{-at}] \end{cases}$$

$$\cdots\cdots$$

$$\begin{cases} x_n(t) = x_{n-1}(t) + x_{n-1}(T)e^{-at} = f(1+e^{-aT} + \cdots + e^{-a(n-1)T})e^{-at} = f\dfrac{1-e^{naT}}{1-e^{-aT}}e^{-at} \\ y_n(t) = y_{n-1}(t) + y_{n-1}(T)e^{-at} = \dfrac{af}{a-b}(\dfrac{1-e^{nbT}}{1-e^{-bT}}e^{-bt} - \dfrac{1-e^{naT}}{1-e^{-aT}}e^{-at}) \end{cases}$$

$$\lim_{n\to\infty} y_n(t) = \frac{af}{a-b}(\frac{e^{-bt}}{1-e^{-bT}} - \frac{e^{-at}}{1-e^{-aT}})$$

So the alcohol content in body fluid is shown in Equation (4).

$$c(t) = \frac{\lim_{n\to\infty} y_n(t)}{v} = \frac{af}{v(a-b)}(\frac{e^{-bt}}{1-e^{-bT}} - \frac{e^{-at}}{1-e^{-aT}}) \quad (4)$$

3. The Experiment and Application of Models

The following data can be obtained through experimental tests. Human body fluid accounts for 65% to 70% of human body weight, of which blood accounts for only about 7% of body weight. The experiment shows that when someone weighing about 70 kg drank two bottles of beer in a short time, the alcohol content in his blood (mg/100 ml) measured at a certain time is shown in Table 1.

Table 1 The alcohol content in his blood (mg/100 ml)

Time(hour)	0.25	0.5	0.75	1	1.5	2	2.5	3	3.5	4	4.5	5
Alcohol content	30	68	75	82	82	77	68	68	58	51	50	41
Time (hour)	5	6	7	8	9	10	11	12	13	14	15	16
Alcohol content	41	38	35	28	25	18	15	12	10	7	7	4

The experimental data shows that 1 bottle of beer is 650 ml, the alcohol content is 4%. From $c(t) = \dfrac{y(t)}{v} = \dfrac{af}{v(a-b)}(e^{-bt} - e^{-at})$ and Table 1, the parameters $a = 2.026$ and $b = 0.154$ are obtained by the least square method and MATLAB software.

According to national standards, the alcohol content in human body fluid $c(t) < 20$ is safe for driving. Then $c(t) < 20$. Combining Equation (1), (3), and (4) obtained in previous solutions, to scientifically calculate. If planning to drive after t hours without violating the regulations, the amount of alcohol f every day can be calculated; if the amount of alcohol is f, it is safe to drive after t hours.

Taking Equation (4) as an example, for a person weighting about 70 kg, $T=24$, $a=2.026$, $b=0.154$, $v=70\cdot 65\%\cdot 7\%=3.185$, $c(t)<20$. Then the amount of alcohol and safe time waiting for driving can be calculated and shown in Table 2.

Table 2 The amount of alcohol and waiting time

The amount of drinking (bottles of beer)	0.4894	0.5713	1	2	3
Waiting time (hour)	2	3	6	10	13

From Table 2, drivers can determine how many hours after drinking to drive safely based on how much they drink. If drink 0.4894 bottles of beer, drive safely after 2 hours; if drink 0.5713 bottles of beer, drive safely after 3 hours; if drink 1 bottle of beer, drive safely after 6 hours; if drink 2 bottles of beer, drive safely after 10 hours; if drink 3 bottles of beer, drive safely after 13 hours.

Drivers can also decide how much to drink for safe drive according to their own schedule. If want to drive after 2 hours of drinking, drink no more than 0.4894 bottles of beer; if want to drive after 3 hours, drink no more than 0.5713 bottles of beer; if want to drink after 6 hours of drinking, drink not exceed 1 bottle of beer; if want to drive after 10 hours of drinking, drink no more than 2 bottles of beer; if want to drive 13 hours after drinking, drink no more than 3 bottles of beer.

4. Conclusion

In this paper, according to the changes of alcohol content in blood after drinking alcohol, mathematical model is established after mathematical and medical analysis. The equation $c(t)$ showing changes of alcohol content in blood is concluded for people drink in different modes (drinking for a short time and for a long time). Therefore, people can choose the corresponding equation $c(t)$ according to their actual drinking situation, to scientifically calculate the time for safe driving after drinking or to scientifically calculate the amount of alcohol according to their schedule.

Acknowledgement

Fund project: This work is fund project of Education Department of Shaanxi (17JK1034); fund project of key curriculum construction of Xi'an Eurasia University (2018KC032); project of Xi'an Eurasia University (2018YB011).

References

[1] Wu Qixing. Why the Death Rate of Traffic Accidents is low in America? http://www.chinahighway.com/news/2007/209276.php, eastday.com, 2013-3-3.

[2] Xiong Yongming. Necessity of Make Acts of Drunk Driving into Criminal [J]. Journal of Shanxi Politics and Law Institute for Administrators, 2010, 23(1): 96-97.

[3] Xu Wei, Lin Yan. On the Frequently-occurred Drunk-driving under Strict Check and the Necessity of Adding Crime of Dangerous Driving [N]. Jurisprudence Daily, 2009-9-24.

[4] Band C of 2004 China Undergraduate Mathematical Contest in Modeling [EB/OL] http://www.mcm.edu.cn.2004-09-17.

[5] Zhu Yiyi. Analysis on Relevant Problems of Crime of Dangerous Driving for Drunk-driving from the Perspective of Risk Society [D]. East China University of Political Science and Law, 2017. 10.

[6] Richard J. Stringer. Exploring traffic safety culture and drunk driving: An examination of the community and DUI related fatal crashes in the U.S. (1993–2015) [J]. Transportation Research Part

F: Psychology and Behaviour, 2018 (14), 371-380.

[7] Ted R Miller; David T. Levy; David I. Swedler. Lives saved by laws and regulations that resulted from the Bloomberg road safety program [J]. Accident Analysis and Prevention, 2018, 131-136.